TECHNICAL NOTE
RELATIONSHIP BETWEEN FIGHTER ACCELERATION
AND ENERGY MANEUVERABILITY

Aircraft velocity in the wind axis coordinate system is described in equation (1).

$$\vec{v} = v\,\vec{i}_w \tag{1}$$

To get acceleration, we take the time derivative of equation (1) as in equations (2) and (3).

$$\vec{a} = \frac{d\vec{v}}{dt} = \dot{v}\,\vec{i}_w + v\,\frac{d\vec{i}_w}{dt} \tag{2}$$

$$\vec{a} = \dot{v}\,\vec{i}_w + vr_w\,\vec{j}_w - vq_w\,\vec{k}_w \tag{3}$$

We can write equation (3) in different ways. In one case, we can derive the coefficients of the unit vectors from the following three equations of motion.

$$T - D - mg\sin\gamma = m\dot{v} \tag{4}$$

$$mg\sin\mu\cos\gamma = mv\,r_w \tag{5}$$

$$L - mg\cos\mu\cos\gamma = mv\,q_w \tag{6}$$

For the coefficient along the flight path, we solve equation (4) and put it in energy maneuverability terms as shown in equations (7).

$$\begin{aligned}\dot{v} &= \frac{T - D}{m} - g\sin\gamma \\ &= g\left(\frac{P_s}{v} - \sin\gamma\right)\end{aligned} \tag{7}$$

Solving equation (5) for $vr_w$ and equation (6) for $vq_w$, we get equations (9) and (10) by using equation (8).

$$\begin{aligned}L\cos\mu\cos\gamma &= W \\ n &= \frac{L}{W}\end{aligned} \tag{8}$$

Finally, we use equations (7), (9) and (10) to get equation (11), a representation of

$$vr_w = g \sin\mu \cos\gamma = \frac{g\sqrt{n^2\cos\gamma^2 - 1}}{n} \tag{9}$$

$$vq_w = \frac{L}{m} - g \cos\mu \cos\gamma = g(\frac{n^2 - 1}{n}) \tag{10}$$

equation (3) in $P_s$ and load factor form.

$$\vec{a} = g(\frac{P_s}{v} - \sin\gamma)\ \vec{i} + \frac{g\sqrt{n^2\cos\gamma^2 - 1}}{n}\ \vec{j}_w + g(\frac{n^2 - 1}{n})\ \vec{k}_w \tag{11}$$

We also can relate $r_w$ and $q_w$ (the rotational components around the j and k wind axes) to the bank, climb and heading angles $\mu$, $\gamma$ and $\chi$, respectively, as in equations (12).

$$\begin{aligned} q_w &= \dot{\gamma}\cos\mu + \dot{\chi}\sin\mu\cos\gamma \\ r_w &= -\dot{\gamma}\sin\mu + \dot{\chi}\cos\mu\cos\gamma \end{aligned} \tag{12}$$

Substituting equations (7) and (12) into equation (3) provides equation (13) in $P_s$ and rotational terms.

$$\vec{a} = g(\frac{P_s}{v} - \sin\gamma)\vec{i}_w + v(-\dot{\gamma}\sin\mu + \dot{\chi}\cos\mu\cos\gamma)\vec{j}_w + v(\dot{\gamma}\cos\mu + \dot{\chi}\sin\mu\cos\gamma)\vec{k}_w \tag{13}$$

Equations (11) and (13) demonstrate standard energy maneuverability terminology along the wind axis only because neither of the other two components is directed at the center of curvature of the flight path, the point around which aircraft turn rate normally is described. To set up such a systen, the derivative of (1) taken along the flight path also can be derived in a "two-vector system" as in equation (14) where r is the radius of curvature of the flight path.

$$\vec{a} = \frac{d\vec{v}}{dt} = \dot{v}\ \vec{i}_w + \frac{v^2}{r}\ \vec{i}_n + (o)\ \vec{i}_p \tag{14}$$

This representation, with vectors along the wind and normal axes (and with binormal vector equal to zero), simply acknowledges that any vector can be resolved into a perpindicular coordinate system in which one of the components is zero.

The plane formed by the two vectors is called the osculating plane. The axis system sometimes is called the Frenet system. The normal component (n) always points to the center of curvature of the flight path. The osculating plane continually rotates about the wind axis to maintain the binormal component (p) at a zero value.

We find the coefficient of the normal vector in terms of the wind axis coefficients as shown in equation (15).

$$\vec{a} = \frac{d\vec{v}}{dt} = \dot{v}\,\vec{i}_w + \sqrt{(vr_w)^2 + (vq_w)^2}\,\vec{i}_n + (o)\,\vec{i}_p \tag{15}$$

By using the relationships from equations (9), (10) and (12) to derive the right hand sides of equations (16) and (17), we get three forms for acceleration as in equations (18). This represents acceleration in energy maneuverability terms.

$$\sqrt{(vr_w)^2 + (vq_w)^2} = v\sqrt{\dot{\chi}^2\cos^2\gamma + \dot{\gamma}^2} \tag{16}$$

$$\sqrt{(vr_w)^2 + (vq_w)^2} = g\sqrt{n^2 + \cos\gamma^2 - 2} \tag{17}$$

$$\begin{aligned} \vec{a} &= g(\frac{P_s}{v} - \sin\gamma)\,\vec{i}_w + \frac{v^2}{r}\,\vec{i}_n \\ &= g(\frac{P_s}{v} - \sin\gamma)\,\vec{i}_w + v\sqrt{(\dot{\chi})^2\cos\gamma^2 + (\dot{\gamma})^2}\,\vec{i}_n \\ &= g(\frac{P_s}{v} - \sin\gamma)\,\vec{i}_w + g\sqrt{n^2 + \cos\gamma^2 - 2}\,\vec{i}_n \end{aligned} \tag{18}$$

By setting the climb angle ($\gamma$) to zero and adopting standard turn rate terminology from equation (19), equations (20) become descriptions of acceleration in more familiar energy maneuverability terms.

$$\dot{\omega} = \sqrt{\dot{\chi}^2\cos^2\gamma + \dot{\gamma}^2} \tag{19}$$

$$\begin{aligned} \vec{a} &= g\frac{P_s}{v}\,\vec{i}_w + \frac{v^2}{r}\,\vec{i}_n \\ &= g\frac{P_s}{v}\,\vec{i}_w + v\dot{\omega}\,\vec{i}_n \\ &= g\frac{P_s}{v}\,\vec{i}_w + g\sqrt{n^2 - 1}\,\vec{i}_n \end{aligned} \tag{20}$$

References

(1) Miele, Flight Mechanics, Theory of Flight Paths, Addison-Wesley Publishing Company, Inc., Massachusetts, 1962.

(2) Boyd, John R. and others, Energy Maneuverability, Technical Report APGC-TR-66-4, Volume 1, Confidential, Eglin Air Force Base, Florida, 1966.

(3) Boyd, John R. and others, Maximum Maneuver Concept, Technical Report ATATL-TR-72-116, Unclassified, Eglin Air Force Base, Florida, 1972.

# DERIVATION OF AGILITY PARAMETERS

An important body of analytical work on fighter agility was produced in 1988 by Messerschmitt-Bölkom-Blohm (MBB) and published under a Rockwell International report on the EFM/X-31A program. This paper summarizes some of that material and suggests a way to use it to insert the values of agility metrics (measured by flight tests or six-degree-of-freedom aircraft models) into an energy maneuverability framework.

The starting point is from equation (14) in the energy maneuverability paper.

$$\vec{a} = \frac{d\vec{v}}{dt} = \dot{v}\,\vec{i}_w + \frac{v^2}{r}\,\vec{i}_n + (o)\,\vec{i}_p \tag{1}$$

In the MBB paper, the predominant view of fighter agility was derived by Dr. M. K. Horn (MBB). He suggested that agility could be defined as the time derivative of aircraft acceleration, a kinematic quantity that in earlier days was called "jump." That term will be used here. Aircraft jump (J), then, is defined by the derivative of equation (1).

$$\vec{J} = \frac{d(\vec{a})}{dt} = \frac{d(\dot{v}\,\vec{i}_w)}{dt} + \frac{d(\frac{v^2}{r}\,\vec{i}_n)}{dt} + \frac{d((o)\,\vec{i}_p)}{dt} \tag{2}$$

To take the derivatives in equation 2, we need a vector describing the rotation of the coordinate system with respect to a fixed reference system. We can start with coordinate transformations such as those found in Miele (Reference 1), pages 46-48. From those, using the subscript **w** to define vectors in the aircraft wind axis system, we can write a rotational vector as shown in equation (3).

$$\begin{aligned}\vec{\omega}_w = {} & (\dot{\mu} - \dot{\chi}\sin\gamma)\,\vec{i}_w + (\dot{\gamma}\cos\mu + \dot{\chi}\sin\mu\cos\gamma)\,\vec{j}_w \\ & + (-\dot{\gamma}\sin\mu + \dot{\chi}\cos\mu\cos\gamma)\,\vec{k}_w\end{aligned} \tag{3}$$

To convert this to the coordinate system of equation (1), we can make use of the following ideas. In equation (1)'s coordinate system, the **p** component of acceleration is always zero because the aircraft motion vector is resolved continually into the osculating plane. To determine the conditions for this resolution, we can use the wind-axis acceleration equation (equation (13) in the energy maneuverability paper) and choose **μ** (bank angle) such that the **p** component vanishes. We also need another label for this rotation about the velocity vector since **μ** is normally reserved for the angle of inclination of the aircraft lift vector. In this paper, we use the capitalized version of **μ**, **M**, to represent the inclination of the osculating plane. With some algebra, we find equations (4).

$$\tan\mu = -\frac{\dot{\gamma}}{\dot{\chi}\cos\gamma} \tag{4}$$
$$-\dot{\gamma}\sin\mu + \dot{\chi}\cos\mu\cos\gamma = \dot{\omega}$$
$$\dot{\mu} = \dot{\mathrm{M}}$$

In equations (4), the rotational term, $\dot{\omega}$, is the same as previously defined in equation (19) of the energy-maneuverability paper. Putting these results into equation (3), yields the rotation vector we need in the coordinate system being used. The result is shown below.

$$\vec{\omega} = (\dot{\mathrm{M}} - \dot{\chi}\sin\gamma)\,\vec{i}_w + (0)\,\vec{i}_n + \omega\,\vec{i}_p \tag{5}$$

Using the rotational vector in the Poisson equations (see Miele) for taking the derivatives of the unit vectors in equation (2), then, we get the following equation for agility (A).

$$\vec{A} = (\ddot{v} - \omega^2 v)\,\vec{i}_w + (2\,\dot{v}\,\omega + v\,\dot{\omega})\,\vec{i}_n + v\,\omega\,(\dot{\mathrm{M}} - \dot{\chi}\sin\gamma)\,\vec{i}_p \tag{6}$$